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Unmodeled Dynamics and Data Driven Balance Control for a Class of Underactuated Mechanical Systems*

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***Abstract*—Unmodeled dynamics and data driven balance control strategy are presented in this paper for a class of underactuated mechanical systems with two freedoms. The idea behind the method are as follows. First, the underactuated system is divided into two subsystems. Linear models for each subsystem are constructed from the experimental datum. The proportional derivative (PD) controller can be designed by this linear model. The unmodeled dynamic compensator is designed to deal with modelling error between the linear model and the real model. Second, to control two outputs of the underactuated systems with one input at the same time, a coordinative control scheme is introduced for weighting the control inputs of the two subsystems. Finally, in order to value the proposed control scheme, the Pendubot (pendulum robot) is selected as the experimental platform to verify the method. Experimental results show that the proposed control strategy can be easily applied and has higher control precision than the existing methods.**

***Keywords*—underactuated manipulator, balance control, unmodeled dynami, PD control**

I. INTRODUCTION

Underactuated systems are mechanical control systems with smaller number of controls (actuators) than the configuration variables (degree of freedoms) [1]. The study of underactuated systems has always been a field of active research, due to their practical applications in robotics, aerospace and marine vehicles. Pendubot (pendulum robot), is a common example for underactuated systems. Different from the linear inverted pendulum or the rotational inverted pendulum, the Taylor series linearized model of Pendubot computed around any operating point changes at each operating point [2]. Therefore, the Pendubot possesses many attractive features and has been a classical and contemporary benchmark for testing different control algorithms.

Many papers on the balance control of Pendubot systems have been published such as sliding-mode control [4] and LQR (linear quadratic regulator) control [4]. However, these strategies depend on an accurate plant model. Intelligent control does not need the model of the systems such as fuzzy control [5], T-S Fuzzy control [6], genetic algorithms [7], neuro-fuzzy controller [8], and so on. Instead, these strategies always need more experience for controller design. To deal the above problems, this paper proposes unmodeled dynamics and data driven balance control scheme without knowing about its exact structure and parameters. This strategy not only can greatly improve the steady-state precision of the system and effectively eliminate the disturbance from unmodeled dynamics, uncertainty and friction, but also can be easily applied and has higher performance than the existing methods.

II. PROBLEM STATEMENT

The model of Pendobot can be described by Lagrange formulation as follows [9] [10]:

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F=U \tag{1}$$

where $q=[q_1\ q_2]^{\mathrm{T}}\in R^2$ denotes the arm angle of Pendubot, and $\dot{q}=[\dot{q}_1\ \dot{q}_2]^{\mathrm{T}}\in R^2$ denotes the arm angular velocity of the Pendubot, $U=[u\ 0]^{\mathrm{T}}\in R^2$ is the applied torque input vector, $D(q)\in R^{2\times 2}$ is the symmetric positive define Pendubot inertia matrix, $C(q,\dot{q})\dot{q}\in R^2$ is the vector of centripetal and coriolis torque, and $G(q)\in R^2$ stands for the vector of gravitational torques due to the gravity, $F=[f_d\ 0]^{\mathrm{T}}\in R^2$ is friction torque vector.

Let $x_1=q_1$, $x_2=\dot{q}_1$, $x_3=q_2$, $x_4=\dot{q}_2$, $y_1=q_1$, $y_2=q_2$ then (1) can be described as follows:

$$\begin{aligned}&\dot{x}_1=x_2\\&\dot{x}_2=f_1(x)+b_1(x)u+c_1(x)f_d\\&\dot{x}_3=x_4\\&\dot{x}_4=f_2(x)+b_2(x)u+c_2(x)f_d\\&y=[x_1,\ x_3]^T\end{aligned} \tag{2}$$

where

$f_1(x)=-[1\ 0]D^{-1}(q)[C(q,\dot{q})\dot{q}+G(q)+F]$,

$f_2(x)=-[0\ 1]D^{-1}(q)[C(q,\dot{q})\dot{q}+G(q)+F]$,

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$b_1(x) = -[1\ 0]D^{-1}(q)[1\ 0]^T$, $c_1(x) = -[1\ 0]D^{-1}(q)[1\ 0]^T$,
$b_2(x) = [0\ 1]D^{-1}(q)[1\ 0]^T$, $c_2(x) = -[0\ 1]D^{-1}(q)[1\ 0]^T$.

Equation (2) shows that the underactuated system is divided into two subsystems. This form can be treated as the norm expression of a class of underactuated systems [1]. The goal of balance control is to design controller to realize $q_1 = 90°$ and $q_2 = 0°$ without knowing the knowledge on the exact structures and parameters of model.

III. Unmodelled Dynamics and Data Driven Controller Design for Pendubot Systems

To begin with, a model for controller design is constructed from the experimental datum of the Pendubot systems. And then, the unmodeled dynamics compensation and data driven balance control strategy for Pendubot are introduced.

A. Model for Controller Design

As shown in (2), the model of Pendubot can be described as two subsystems. We can get the discrete models of the system by system identification method:

$$A_1(z^{-1})y_1(k+1) = B_1(z^{-1})u(k) + V_1[x(k)] \tag{3}$$

$$A_2(z^{-1})y_2(k+1) = B_2(z^{-1})u(k) + V_2[x(k)] \tag{4}$$

where the $u(k)$, $y_1(k)$ and $y_2(k)$ are the input and outputs of the system; $A_i(z^{-1}) = 1 + a_{i1}z^{-1} + a_{i2}z^{-2}$, $B_i(z^{-1}) = b_{i0} + b_{i1}z^{-1}$, $i = 1,2$; $a_{i1}, a_{i2}, b_{i0}, b_{i1}$ are system parameters; z^{-1} is a backward operator; $V_1[x(k)]$ and $V_2[x(k)]$ are the unmodeled dynamics of the system which include the nonlinear terms and the mutual influence of each subsystem.

Then it can be obtained that

$$V_1[x(k)] = A_1(z^{-1})y_1(k+1) - B_1(z^{-1})u_1(k) \tag{5}$$

$$V_2[x(k)] = A_2(z^{-1})y_2(k+1) - B_2(z^{-1})u_2(k) \tag{6}$$

B. Design of Controller

The diagram of the balance controller with unmodeled dynamics compensation is as Fig.1.

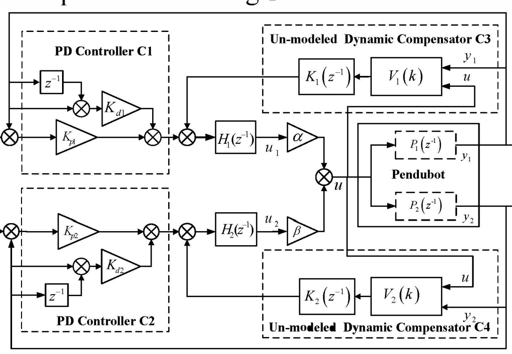

Fig.1. Diagram of the balance controller with un-modeled dynamics compensator

Based on the above-mentioned subsystem model, the main design idea is as follows: firstly, based on the linear controls to design the PD controllers and unmodelled dynamics compensators of subsystem P_1 and P_2; then to compute the control input by weighting the output of two controllers for the two subsystems.

As for subsystem P_1 and P_2 have the same structure, we design PD controller C1 and unmodeled dynamic compensator C3 for P_1 firstly. The structure of the PD controller with unmodeled dynamics compensation for subbsystem P_1 is:

$$H_1(z^{-1})u_1(k) = K_{p1}e_1(k) + K_{d1}[e_1(k) - e_1(k-1)] - K_1(z^{-1})V_1[x(k)] \tag{7}$$

where the K_{p1} and K_{d1} are the proportional and differential gains of PD controller; $e_1(k) = w_1(k) - y_1(k)$; $w_1(k)$ denotes the expected output; $y_1(k)$ is the output of the system; the filter is $H(z^{-1}) = 1 + h_1 z^{-1}$, h_1 is the undetermined coefficient; $V_1[x(k)]$ denotes nonlinear compensation; $K_1(z^{-1})$ is the compensation polynomial.

Substituting the controller (7) into the system (3), we can get the closed-loop system equation:

$$\begin{aligned}&[A_1(z^{-1})H_1(z^{-1}) + z^{-1}B_1(z^{-1})G_1(z^{-1})]y_1(k+1) = \\ &B_1(z^{-1})G_1(z^{-1})w_1(k) + [H_1(z^{-1}) - B_1(z^{-1})K_1(z^{-1})]V_1[x(k)]\end{aligned} \tag{8}$$

where $G_1(z^{-1}) = g_0 + g_1 z^{-1}$, $g_0 = K_{p1} + K_{d1}$, $g_1 = -K_{d1}$.

Then we can get the coefficient of PD controller as follows:

$$\begin{cases} K_{p1} = g_0 + g_1 \\ K_{d1} = -g_1 \end{cases} \tag{9}$$

In order to achieve better dynamic performance, we apply the pole-placement method to realize the placement of the closed-loop poles. We assume that the closed-loop characteristic polynomial is $T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-2}$. Then we have

$$A_1(z^{-1})H_1(z^{-1}) + z^{-1}B_1(z^{-1})G_1(z^{-1}) = T(z^{-1}) \tag{10}$$

The parameters of $T(z^{-1})$ can be determined by $s^2 + 2\xi w_n s + w_n^2$, then we get

$$t_1 = -2\exp(-\zeta\omega_n T_0)\cos(\omega_n T_0\sqrt{1-\zeta^2}) \tag{11}$$

$$t_2 = \exp(-2\zeta\omega_n T_0) \tag{12}$$

where T_0 is the sampling period; ζ is damping coefficient, w_n is the natural mode shape.

The influence of unmodeled dynamics on the system performance can be offset by constructing appropriate weighted polynomial $K_1(z^{-1})$. We let $H_1(z^{-1}) - B_1(z^{-1})K_1(z^{-1}) = 0$ when $z \to 1$.That is to say, $K_1(1) = H_1(1)/B_1(1)$ can eliminate the influences of the unmodeled dynamics on the performance of the closed-loop system.

Considering the unmodeled dynamic $V_1[x(k)]$ of P_1 is unknown, but $V_1[x(k-1)]$ is easy to get. In order to

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compensate the influence of the unmodeled dynamic to select the controller design structure as:

$$\begin{aligned} H_1(z^{-1})u_1(k) = K_{p1}e_1(k) + K_{d1}[e_1(k) - e_1(k-1)] \\ - K_1(z^{-1})V_1[x(k-1)] \end{aligned} \quad (13)$$

Design of PD controller C2 and unmodeled dynamics compensator C4 for subsystem P_2 is similar with that of subsystem P_1. The controller for subsystem P_2 is designed as:

$$\begin{aligned} H_2(z^{-1})u_2(k) = K_{p2}e_2(k) + K_{d2}[e_2(k) - e_2(k-1)] \\ - K_2(z^{-1})V_2[x(k-1)] \end{aligned} \quad (14)$$

According to the above-mentioned method of controller design, we design separately the controller for subsystem P_1 and P_2. To control two outputs of the underactuated systems with one input at the same time, a coordinative control scheme is introduced and shown as:

$$u(k) = \alpha u_1(k) + \beta u_2(k)$$

where $u_1(k)$, $u_2(k)$ are the controller output of subsystem P_1 and P_2; $u(k)$ is the control input for Pendubot; α, β are the weighting coefficient and $\alpha > 0$, $\beta > 0$, $\alpha + \beta = 1$.

IV. EXPERIMENTAL RESULTS

In this paper, we use the Pendubot produced by the NDD Intelligent Technology Company as the experimental system. This system including hardware and software combines experimental apparatus with easy-to-use software platform based on matlab/simulink, the network control connection of Pendubot system is given in Fig.2, which includes PC, embedded controller and the Pendubot.

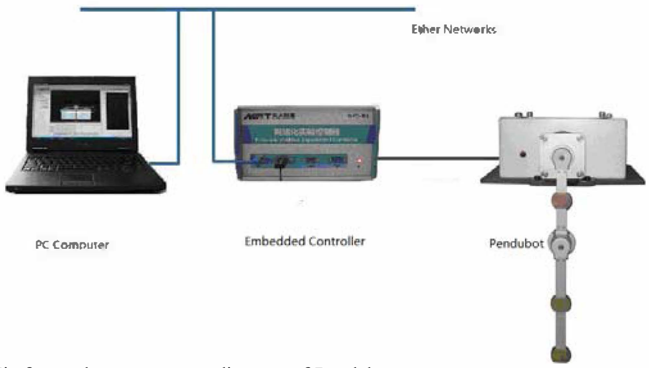

Fig.2. Hardware structure diagram of Pendubot system

A. Controller Parameter Design

To begin with, we set the sampling period as 2ms, collect the experimental datum, and identify the system parameters and. The discrete model of the system at the top-equilibrium point is as follows:

$$A_1(z^{-1}) = 1 - 1.999z^{-1} + 0.999z^{-2},$$

$$B_1(z^{-1}) = 8.86 \times 10^{-5}(z^{-1} + 1.107z^{-2}),$$

$$A_2(z^{-1}) = 1 - 1.999z^{-1} + 0.999z^{-2},$$

$$B_2(z^{-1}) = -1.9339 \times 10^{-6}(z^{-1} + 42.01z^{-2}).$$

In order to stabilize the closed-loop system, we select $\xi = 1$, $\omega_n = 30$. According to (11) and (12), we have $T(z^{-1}) = 1 - 1.883z^{-1} + 0.887z^{-2}$. We can get $H_1(z^{-1}) = H_2(z^{-1}) = 1 - 0.8521z^{-1}$ by computing (10). Finally, the parameter of PD controller can be designed by (9) and (10), $K_{p1} = -18.9698$, $K_{d1} = -3.5636$, $K_{p2} = -18.9698$, $K_{d2} = -3.5636$. The parameter of unmodeled dynamics compensator is $K_1 = 883$ and $K_2 = 1781$. The weighting parameter can be chosen as $\alpha = \beta = 0.5$.

In order to evaluate the performance of the balance controllers, the LQR, PD and PD with unmodeled dynamic compensator (UDCPD) presented in this paper are chosen under the same conditions. The swing-up controller uses the conventional partial feedback linearization controller. Finally, the paper analyzes the performance of the three algorithms.

B. Experiment Results of UDCPD Control

The output curve of q_1 and q_2 of unmodeled dynamics compensation balance controller proposed in this paper are shown in Fig.3. The curves of control input and unmodeled dynamics are given in Fig.4 and Fig.5.

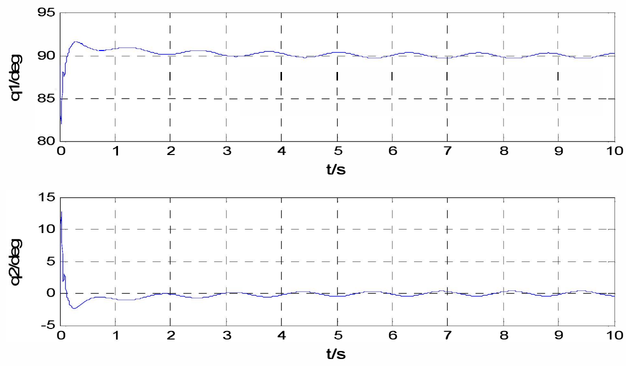

Fig.3. Output angles of UDCPD controller proposed in this paper

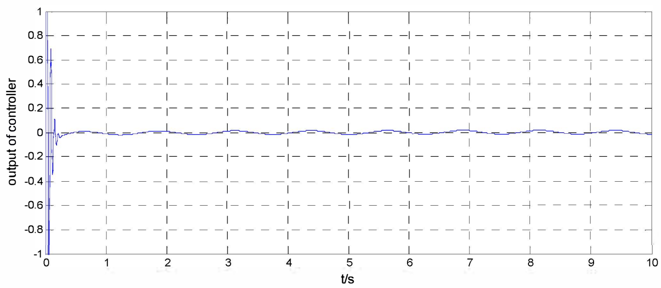

Fig.4. Output $u(k)$ of UDCPD controller proposed in this paper

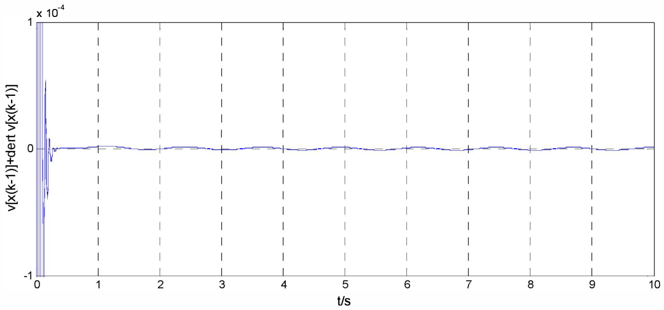

Fig.5. Unmodel dynamics of UDCPD controller proposed in this paper

C. *Experiment Results of LQR Control*

The output curves of q_1 and q_2 of LQR controller are shown in Fig.6.

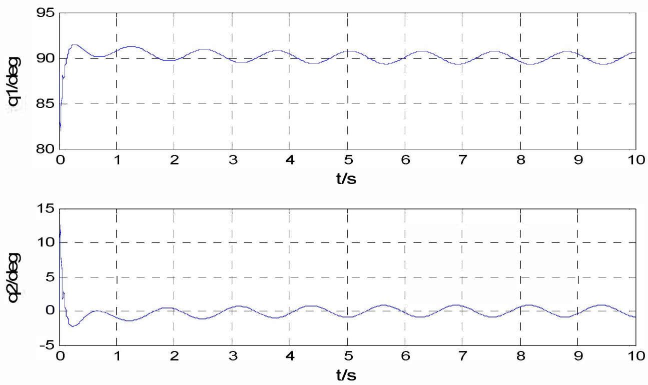

Fig.6. Output angles of LQR controller

D. *Experiment Results of PD Control*

The output curves of q_1 and q_2 of PD controller are shown in Fig.7.

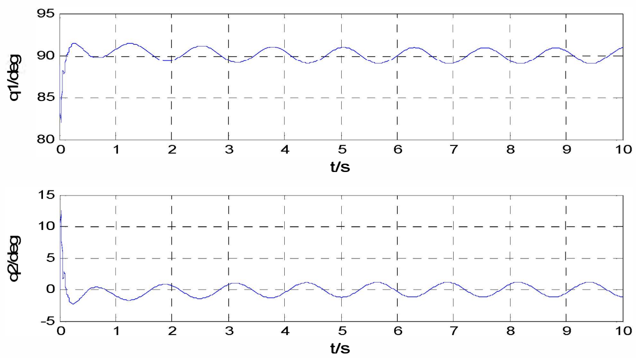

Fig.7. Output angles of PD controller

Table 1 Performance Index

Performance Index / Controller	Steady-state Error of Link1 (deg)	Steady-state Error of Link2 (deg)
PD	2.38	2.25
LQR	1.65	1.88
UDCPD	0.78	0.75

Three experiments adopted the same swing-up control method called the partial feedback controller. Combined with table 1, we can find that the steady-state error has been decreased and the precision of the system has obviously been improved. Compared with the results of LQR and PD controller, the proposed method UDCPD compensated the influences of unmodeled dynamics effectively.

V. Conclusion

Balance controller with unmodeled dynamics compensator for underactuated mechanical system has been presented. The design of PD controller with unmodeled dynamics compensator is based on the discrete model. We can place the closed-loop poles at the expected position and offset the effects of unmodeled dynamics by introducing compensators. The experimental results show that the proposed control strategy not only can guarantee the stability of the system, but also is easy to implement and is of higher performance.

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